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THE IMPEDANCE OF TELEPHONE RECEIVERS AS AFFECTED BY THE MOTION OF THEIR DIAPHRAGMS.

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I. INTRODUCTION.

THE writers have made a series of measurements of the resistance and inductance of several forms of telephone receivers over a wide range of frequency of current. In the course of the measurements some interesting results have been obtained, which form the subject of this paper.

As the period of the e. m. f. used in the measurements approaches the natural period of the diaphragm, the note emitted by the telephone receiver increases markedly in loudness, and the resistance and inductance of the receiver undergo wide deviations from values obtained when the diaphragm is prevented from vibrating by being damped. That is to say, the motion of the diaphragm has an effect upon the resistance and inductance of the receiver, and this effect grows rapidly as the electrical period approaches the mechanical period.

In the tests to be described, the resistance and the inductance of a given receiver were measured, first with the diaphragm free and sounding, and, second, with the diaphragm damped, or arrested. The values when the diaphragm is free may be called *free values;* the values when the diaphragm is damped may be called *damped values.* The difference obtained by subtracting the damped values from the corresponding free values may be called the *motional values* of resistance, inductance, etc.; since such differences are due to the motion of the diaphragm.

It is found that when the impressed frequency differs widely from the natural frequency of the diaphragm, the motional resistance and inductance are very small. In the neighborhood of resonance, which is often very sharply marked, these motional values become relatively large, and one or both pass through a change of sign, in such a manner that, when the motional impedance for different frequencies is drawn vectorially from a fixed point as origin, all the points given by the observations lie upon a circular graph, which may be called the *mo-*

tional impedance circle, or for brevity, the *circular graph* of the receiver. Different telephone receivers have very different circular graphs, and the circular graph of any given receiver defines its characteristics in various important practical respects, both mechanical and electrical.

Also, from a theoretical standpoint, the reactive influence of the motion of the diaphragm upon the current through the coils of a telephone is interesting from its analogy to the effects of an optical medium upon transmitted light in the neighborhood of an absorption band.

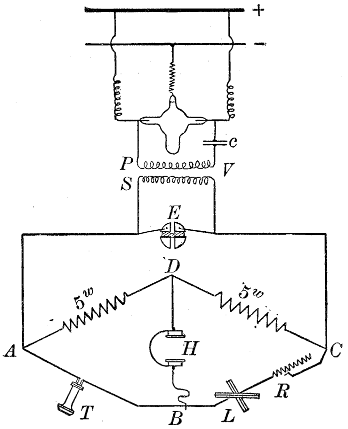

Figure 1. Diagram of connections of the Rayleigh Bridge and Vreeland Oscillator.

The experiments also give information regarding the natural period of vibration, and the natural damping factor of the diaphragm, and the power drawn by the instrument when it is sounding and when it is damped.

II. Method and Apparatus.

Measurement of Resistance and Inductance.— The measurements of resistance and inductance were all made with a Rayleigh equal-arm bridge, with connections as indicated in Figure 1. The arms AD and DC of the Bridge were 5-ohm non-inductive resistances. The Arm AB contained the telephone receiver T under test. The arm BC contained the adjustable non-inductive resistance R and the adjustable Ayrton-Perry Variometer L. A head telephone H served to determine a balance.

The source of e. m. f. used in the measurement was a Vreeland Sine-Wave Oscillator [1] *V*, capable of supplying any desired frequency between the limits 430 and 2400 cycles per second. The secondary coil S of the Vreeland oscillator was connected to the bridge terminals *A* and *C* through a resistance of usually 50 or 100 ohms, not indicated. The potential difference of the Vreeland secondary terminals was kept constant throughout a given set of measurements, and was measured by a Paul static voltmeter E. In order to approximate working conditions in the telephone, the voltage at E was kept down to a low value (6 to 12 volts, differing in different experiments). From the known voltage at E, the voltage at the terminals of the telephone was known, and was maintained in different series of tests at values between 0.3 and 1.0 volt.

Measurement of Frequency.— The frequency of the oscillator was measured by comparing the pitch of the sound from the telephone *T* with that of a set of 75 tonometer tuning forks, ranging from 256 to 552 vibrations per second, with successive differences of 4 vibrations per second. Intermediate pitches were estimated by beats. As a check, a stroboscope was in some instances used upon the flickering illumination from the Vreeland bulb, but this was found to be much less convenient than measurement by the tonometer. The pitches were also obtained by calculation from the electrical constants of the Vreeland oscillation circuit, and this method was found to be the most reliable in interpolations for small changes of frequency. In order to obtain the small gradations of frequency that were sometimes necessary, an auxiliary standard condenser was used in connection with the condensers in the Vreeland oscillation circuit. In certain cases it was necessary to vary the capacity in successive measurements by increments as small as 0.001 microfarad.

Procedure.— The procedure in making a test was as follows: The frequency of the Vreeland oscillator was first set to the required value by the adjustment of the capacity, and occasionally of the inductance, of its oscillation circuit. The magnitude of the voltage at the Vreeland terminals was then adjusted by adjusting the distance between the primary and secondary Vreeland coils, while an observer watched the indications of the Paul static voltmeter *E*. A balance on the Rayleigh Bridge was then obtained by successive adjustments of *R* and *L* (Figure 1) with the diaphragm of the telephone *T* free and sounding. After recording the values of *R* and *L* so obtained, the balance was repeated with the diaphragm of the telephone *T* at rest and silent. The damping was effected usually by lightly pressing upon the diaphragm with the finger, but in some cases it was affected by inserting a light wedge (a quill) between the diaphragm and pole, when this operation was permitted by an open structure telephone. The balance, when the diaphragm was damped, gave practically complete silence in the head-telephones H, and the settings of resistance and inductance were consistent within about ½ of 1%. The balance, on the other hand, when the diaphragm was in motion, was not so good. In this case, difficulties were introduced by parasitic notes probably due to currents of higher frequency generated by the motion of the telephone diaphragm. It was usually possible, however, to balance out the fundamental tone, with adjustments consistent within 1 or 2 ohms.

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[1] Physical Review, **27,** p. 286, 1908.

III. PARTICULARS OF THE TELEPHONES TESTED.

Several telephones were submitted to measurements. Four of the instruments, for which the results are presented in the present account, were: —

1. A Western Electric Bipolar Bell Telephone, Type 122, here designated "R_b",
2. A Western Electric Bipolar Watch-case Telephone receiver, designated "Watch-case,"
3. An experimental specially-constructed monopolar receiver, here designated "Experimental monopolar," and
4. An experimental bipolar telephone receiver, provided with exploring coils, and here designated "Experimental bipolar."

The following table (Table I) contains some of the mechanical particulars of these instruments.

IV. EXPERIMENTAL DATA AND RESULTS.

The data obtained by measurements of the resistance and inductance of the first three of the above receivers are contained in Tables II to VI. The data with the "experimental bipolar" receiver are not tabulated, as they were taken for the specific purpose of determining the angle of lag of magnetization of the iron behind the actuating current and this subject is discussed later.

Explanation of Tables.— A brief explanation of Table II, obtained with the bipolar Bell "R_b" with 0.3 effective volts applied at its terminals, will be given as typical of all the tables. The first column contains the frequency in cycles per second. The second column gives the corresponding angular velocity in radians per second. The third column gives R′ the resistance free, at each frequency, as measured on the Rayleigh bridge; while the fourth column gives **R** the corresponding resistance obtained with the diaphragm damped. The fifth column, headed "motional," gives R′ — R; or the difference between the free and damped resistances with a proper sign for the difference. The three remaining columns contain the corresponding reactances, as obtained by multiplying the inductances observed on the Rayleigh bridge by the angular velocity ω in each case. The final column, marked "motional," gives the excess of free reactance over damped reactance, with proper sign.

TABLE I.

Mechanical Constants of Receivers.

	Bell R_b.	Watch-case.	Exp. Monopolar.	Exp. Bipolar.
Area of each pole in cm. × cm.	1.4 × .225	1.61 × .16	0.53* × 0.53	1.17* × 0.38
Distance separating poles in cm.	0.85	0.80	—	0.73
External diam. of diaphragm in cm.	5.40	5.48	7.22	7.22
Diameter of clamping circle, cm.	4.94	4.84	4.45	4.45
Tchikness of diaphragm, cm.	0.024	0.030	0.032	0.022
Weight of diaphragm, grams	4.0	5.03	5.3	5.3
Direct-current resistance of coils, ohms, at 20° C.	71.0	81.4	89.7	25

* Laminated poles.

Tables III to VI contain data similar to those in Table II, but with different applied voltages or different receivers.

An examination of these tables shows that there are two independent phenomena of interest; namely,

First, the effect of the frequency on the resistance, reactance and inductance of the receiver when damped; and

TABLE II.

Resistance and Reactance of Bipolar Bell Receiver R_b, at Different Frequencies.

0.3 volt at Terminals.

Frequency.		Resistance, Ohms.			Reactance, Ohms.		
n. Cycles per Second.	$\omega = 2\pi n$ Radians per Second.	Free R'.	Damped R.	Motional $R' - R$.	Free $X' = L'\omega$.	Damped $X = L\omega$.	Motional $L'\omega - L\omega$.
440	2760	151.8	146.3	5.5	138.1	135.1	3.0
512	3220	163	150.8	12.2	148.7	144.5	4.2
600	3770	166	147.7	18.3	154.5	150.5	4.0
670	4210	188	157.5	30.5	162.4	158.8	3.6
704	4420	206	163.0	43.0	162.0	164.2	− 2.2
720	4520	213	164	49	158	166	− 8.0
744	4660	233	168	65	142	168.8	−26.8
754	4730	237	170	67	133.2	169.5	−36.3
770	4830	240	172	68	115.7	172.3	−56.6
778	4880	212	173	39	78.1	171.4	−93.3
780	4900	173.5	173	0.5	76.5	170.5	−94.0
790	4960	164	172.5	− 8.5	82.3	173.0	−90.7
792	4970	164	172.5	− 8.5	81.9	173.2	−91.3
792	4970	172.5	183	−10.5	78.4	165.9	−87.5
792	4970	178	175	3	78.4	173.6	−95.2
793	4975	169	175	− 6	78.5	172.9	−94.4
794	4980	178	172.5	5.5	75.6	173.5	−97.9
804	5050	142	178.5	−36.5	122	174	−52.0
808	5080	148	177	−29	98.4	174.9	−76.5
822	5160	146.5	180	−33.5	145	175	−30.0
824	5180	142	180	−38	121.6	180.0	−58.4
824	5180	136	176	−40	120.4	177.2	−56.8
826	5190	141	176	−35	117.7	177.9	−60.2
826	5190	141	176	−35	127.2	177.9	−50.7
832	5220	147	179	−32	145.7	172.9	−27.2
840	5280	157	182	−25	162.8	177.5	−14.7
868	5450	160	183	−23	167	181	−14.0
890	5590	166	186	−20	175	184	− 9.0
892	5600	164.5	185	−20.5	175	184	− 9.0
912	5730	180	204	−24	184	192.5	− 8.5
940	5900	178.7	191.5	−12.8	183.5	187.8	− 4.3
1000	6283	205	203	− 8.0	202.5	204	− 1.5
1060	6650	210	216	− 6.0	209.5	211.5	− 2.0
1152	7250	192	202	−10	230.5	234	− 3.5
1248	7850	222	225	− 3	230	228	2.0
1648	10350	264	265	− 1	254	253	1.0
2464	15500	325	325	0.0	291.5	294	− 2.5

Second, the effect of the motion of the diaphragm. These two effects will be treated in order.

Change of Damped Resistance, Inductance and Reactance with Change of Frequency.— Figure 2 shows the damped resistance, inductance and reactance of the bipolar receiver "R_b" plotted against the angular velocity of the current used in the measurements. Figure 3 contains the corresponding curves for the bipolar "watch-case" receiver. In each case the resistance and reactance of the telephone when damped increases with increase of frequency, while the damped inductance decreases with increase of frequency. The following empirical relations approximately hold.

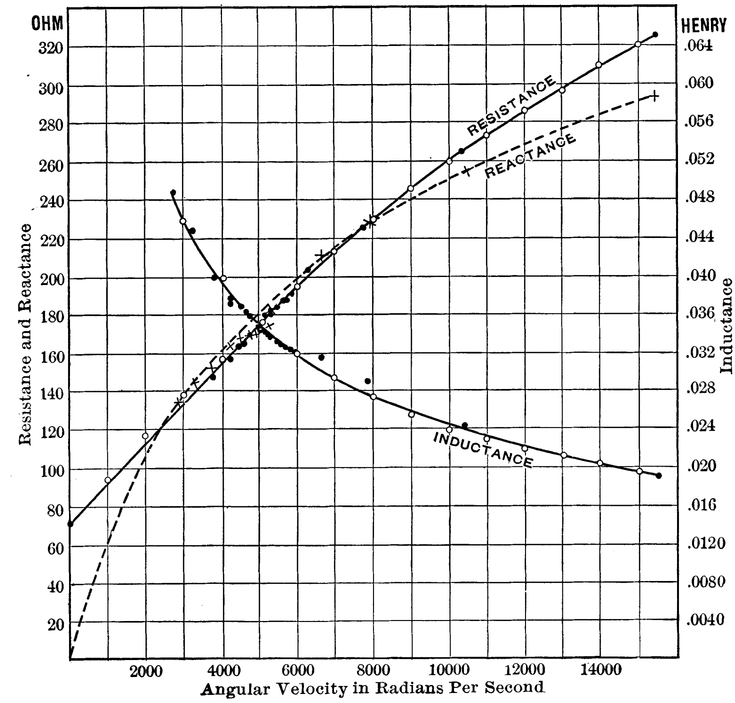

FIGURE 2. Curves of damped resistance, inductance, and reactance plotted against angular velocity, for Bell bipolar receiver, with 0.3 volt at terminals. The dots are observed points; circles calculated; crosses belong to reactance curve.

For the bipolar Bell "R_b," at 20° C, and with 0.3 volts at its termi-

TABLE III.

RESISTANCE AND REACTANCE OF BELL BIPOLAR RECEIVER R_b AT DIFFERENT FREQUENCIES.

0.42 VOLTS AT TERMINALS OF RECEIVER.

Frequency.		Resistance, Ohms.			Reactance, Ohms.		
n. Cycles per Second.	ω Radians per Second.	Free R'.	Damped R.	Motional $R' - R$.	Free $L'\omega$.	Damped $L\omega$.	Motional $L'\omega - L\omega$.
428	2690	138	128.5	9.5	129.5	124.4	5.1
548	3446	162	145.7	16.3	148.8	144.3	4.5
704	4425	211	168	43	164.2	166.4	− 2.2
710	4468	216.5	167.5	49	160.7	166.9	− 6.2
722	4540	224	172	52	155.1	168.7	− 13.6
733	4610	230	170	60	148.7	170.4	− 21.7
744	4680	235	171.5	63.5	144.1	170.4	− 26.3
754	4740	242.5	175	67.5	132.1	171.4	− 39.3
766	4810	237	174	63	120.5	171.6	− 51.1
778	4885	227	175	52	90.8	172.9	− 82.1
792	4976	189	176	13	76.1	177.1	−101.0
805	5060	154	177	−23	94.1	177.5	− 83.4
822	5170	142	181	−39	127.8	179.5	− 51.7
836	5253	143	181	−38	142.9	179.7	− 36.8
853	5354	148	183	−35	156.3	183.1	− 26.8
870	5470	157	185	−28	167.9	184.3	− 16.4
890	5595	161	186	−25	171.5	187.2	− 15.7
2464	15480	306.5	306.5	0.0	291.9	296.5	− 4.6

nals, the damped resistance as a function of the angular velocity is expressible by the equation

$$R = 71 + 0.0234\,\omega - 0.456 \times 10^{-6}\omega^2 \qquad \text{ohms,} \quad (1)$$

in which R is the damped resistance, and ω is the angular velocity in radians per second.

TABLE IV.

RESISTANCE AND REACTANCE OF WATCH-CASE RECEIVER, AT DIFFERENT FREQUENCIES, WITH 0.3 VOLT AT TERMINALS OF RECEIVER.

Frequency.		Resistance, Ohms.			Reactance, Ohms.		
n. Cycles per Second.	ω Radians per Second.	Free R'.	Damped R.	Motional $R' - R$.	Free X'.	Damped X.	Motional $X' - X$.
451	2834	135	136	− 1	115.7	118.5	− 2.8
550	3456	149	149	0	135.8	135.8	0.0
653	4102	163	160	3	150.3	149.5	0.8
702	4410	168	166	2	162.6	160.8	1.8
712	4474	174	171	3	157.4	157.0	0.4
753.5	4738	179	175	4	163.5	163.0	0.5
804	5052	185	181	4	169.5	168.0	1.5
849.5	5340	194	184	10	172.5	174.7	− 2.2
884.3	5554	204	187	17	171.2	178.4	− 7.2
903	5674	212	189	23	160	180.5	−20.5
913	5736	207	190	17	146.1	180.5	−34.4
923	5800	192	191	1	137.8	182.5	−44.7
934	5868	174	192	−18	146	183.5	−37.5
940	5906	169	193	−24	154.4	184.5	−30.1
945	5938	168	193	−25	160.1	184.5	−24.4
957	6014	170	194	−24	172.2	186.0	−13.8
968	6082	173	195	−22	177.6	186.8	− 9.2
980	6158	179	197	−18	182.8	186.5	− 3.7
993	6240	183	198	−15	186.6	188.5	− 1.9
1020	6408	188	200	−12	191.1	191.7	− 0.6
1084	6812	197	206	− 9	197.6	198.3	− 0.7
1157	7270	205	211	− 6	203.1	204.5	1.4
1250	7854	214.5	220	− 5.5	210.6	212.0	1.4
1846	11600	254	256	− 2	261.3	263.0	1.7

For the bipolar "watch-case," at 20° C, and with 0.3 volts applied,

$$R = 81.4 + 0.0214\,\omega - 0.505 \times 10^{-6}\omega^2 \qquad \text{ohms.} \quad (2)$$

In these equations, 71 and 81.4 are the respective resistances of the two instruments to steady currents; the other constants of each equation were determined by using two of the observed points on each of the resistance-frequency curves.

Another more interesting fact, obtainable from the experimental curves, is that the product of the damped resistance by the damped inductance, for each of the two telephones, is approximately a constant independent of the frequency. That is, for the bipolar "R_b," at 0.3 volt,

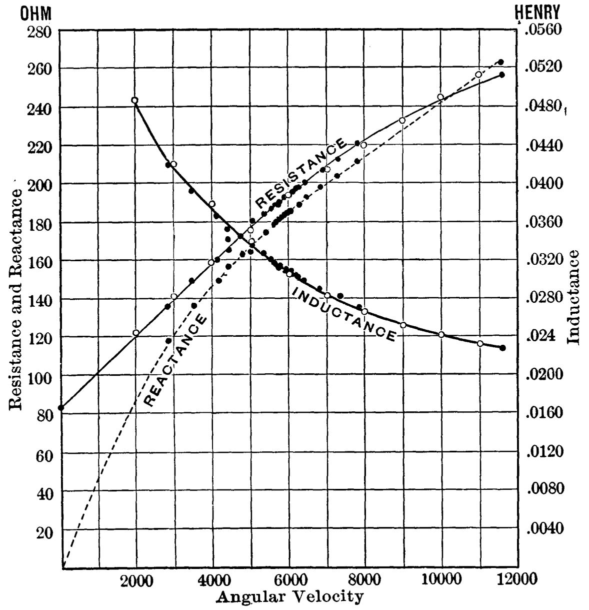

FIGURE 3. Curves of damped resistance, inductance, and reactance *vs.* angular velocity, for watch-case receiver, with 0.3 volt at terminals. Dots, observed; circles calculated.

$$L = \frac{6.25}{R} \qquad \text{henrys} \quad (3)$$

and for the "watch-case" at 0.3 volt

$$L = \frac{5.88}{R} \qquad \text{henrys.} \quad (4)$$

The degree of accuracy with which these formulas accord with the observations is shown by the damped resistance and inductance curves of Figures 2 and 3, where the observed points are indicated by black dots, and the points calculated from the formulas are represented by the circles. The agreement between the calculated points and the curve of observations in the case of the watch-case instrument (Figure 3) is within about 1%. In the case of the Bell instrument "R_b," in order to have sufficient range of frequency, the writers had to use some of the earliest observations, taken before they had learned the precautions required for accurate results. But in this case also, the values calculated by the formulas (1) and (3) agree closely with the curves that best represent the observed points except in regions where the latter are uncertain.

TABLE V.

Resistance and Reactance of Experimental Monopolar Receiver, with 0.3 Volt at Terminals of Receiver.

Frequency.		Resistances.			Reactances.		
n.	$\omega = 2\pi n$.	Free R'.	Damped R.	Motional $R' - R$.	Free X'.	Damped X.	Motional $X' - X$.
993	6240	158	140	18	394	365.3	28.7
1012	6360	321	143	178	354	375.8	− 21.8
1016	6390	266	142	124	427	374.5	52.5
1018	6400	161	142	19	408	378.5	29.5
1020	6410	207	142	65	256	375.8	−119.8
1020	6410	195	143	52	429	378.3	50.7
1023	6428	248	144	104	434	378.2	55.8
1023.6	6436	291	143	148	416	380.6	35.4
1024	6454	211	143	68	432	380.5	51.5
1024	6454	175	143	32	421	382.4	38.6
1024	6435	148	144	4	299	380.0	− 81
1024.2	6440	303	143	160	389	380.0	9.0
1024.8	6444	331	143	188	341	380.4	− 39.4
1025.4	6448	321	143	177	333	380.7	− 47.7
1026	6450	291	143	148	273	380.3	−107.3
1029	6460	238	144	94	257	381.5	−124.5
1032	6480	135	144	− 9	320	380.3	− 60.3
1035	6508	175	144	31	271	383.0	−112
1048	6586	146	144	2	307	387.7	− 80.7
1051	6603	134	144	− 10	340	389.5	− 49.5
1054	6624	140	145	− 5	321	388.7	− 67.7

As a further illustration of the approximate constancy $R \times L$ taken with the telephone damped, reference is made to Table VII, which contains this product at different frequencies for receiver R_b with 0.42

volt at its terminals. At this voltage, the product for this telephone averages 6.21, and within the range of frequencies between 428 and 890 cycles per second, the product does not depart from the average by more than 2%. There is no march of the product within this range.

TABLE VI.

RESISTANCE AND REACTANCE OF EXPERIMENTAL BIPOLAR RECEIVER PROVIDED WITH EXPLORING COILS, WITH 1 VOLT AT TERMINALS.

Frequency.		Resistance, Ohms.			Reactance, Ohms.		
n. Cycles per Second.	ω. Radians per Second.	Free R'.	Damped R.	Motional $R' - R$.	Free X'.	Damped X.	Motional $X' - X$.
1007	6370	39.4	38.3	1.1	153.7	151.8	1.9
1020	6410	42.7	38.6	4.1	156.1	152.9	3.2
1020	6410	41.2	39.2	2.0	155.1	152.5	2.6
1023	6430	44.0	38.7	5.3	156.2	153.0	3.2
1026.5	6450	50.0	38.8	11.2	149.7	153.6	−3.9
1027	6450	42.0	39.0	3.0	156.5	153.6	2.9
1027	6450	48.6	38.8	9.8	153.6	153.9	−0.3
1027	6452	49.0	38.8	10.2	149.7	153.6	−3.9
1027.5	6455	49.0	38.8	10.2	148.0	153.9	−5.9
1028	6458	47.0	39.0	8.0	147.2	153.7	−6.5
1030	6470	43.0	39.0	4.0	157.4	154.1	3.3
1030	6470	48.2	38.8	9.4	148.8	154.0	−5.2
1033	6492	47.2	38.9	8.3	147.4	154.6	−7.2
1033	6492	46.0	39.0	7.0	156.5	154.5	2.0
1033	6492	50.2	38.9	11.3	154.6	154.3	0.3
1036	6510	44.8	39.0	5.8	158.2	154.9	3.3
1036	6510	46.0	39.0	7.0	148.6	155.2	−6.6
1037	6520	46.0	39.0	7.0	150.0	155.3	−5.3
1038	6525	44.0	39.0	5.0	148.4	155.6	−7.2
1039	6530	50.0	39.0	11.0	151.7	154.6	−3.9
1039	6530	44.0	39.0	5.0	149.1	156.0	−6.9
1039.5	6533	43.0	39.0	4.0	148.4	155.6	−7.2
1041	6540	42.0	39.0	3.0	148.5	155.7	−7.2
1042	6546	48.0	39.0	9.0	149.8	155.7	−5.9
1042	6546	40.2	39.1	1.1	149.3	155.9	−6.6
1043	6560	41.0	39.0	2.0	150.0	156.6	−6.6
1045	6570	40.0	39.0	1.0	150.5	156.8	−6.3
1047	6580	39.0	39.0	0.0	150.7	156.7	−6.0
1051	6600	39.9	39.3	0.6	151.3	157.2	−5.9
1051	6604	38.9	39.3	− 0.4	152.5	156.8	−4.3
1054	6624	39.0	39.3	− 0.3	153.0	158.0	−5.0
1059	6660	38.8	39.3	− 0.5	155.2	158.5	−3.3
1064	6686	38.8	39.6	− 0.8	155.2	159.2	−4.0
1084	6812	39.0	39.6	− 0.6	159.0	161.7	−2.7

When, however, the computation is extended to a single observation at 2464 cycles per second, a departure of 5% is obtained.

A third interesting fact shown by the experimental tables is that the damped reactance is approximately equal to the damped resistance for the telephone "R_b" over a wide range of frequency. This may be seen, for this telephone, at 0.3 volt, by a reference to Figure 2 and by a comparison of the observed reactance values, marked by *crosses*, with the observed points on the resistance curve marked by black *dots*. The damped reactances and damped resistances are seen to be nearly the same throughout the range of frequencies between 451 cycles per second ($\omega = 2834$) and 1250 cycles per second ($\omega = 7850$). Within this range the damped resistance and the damped reactance both nearly double and yet remain within a few percent of equality with each other. For the telephone R_b at 0.42 volts, the same approximate equality holds within the range of frequencies between 428 and 2464 cycles per second, as may be seen by a reference to the fourth and seventh columns of Table III. It is to be noted, however, that this same equality cannot persist at low frequencies, for the damped reactance at zero cycles is zero, while the resistance of this instrument at zero cycles is 71 ohms. As a corollary, it may be observed that within the range of equality of damped resistance and damped reactance, the damped angle of lag of current behind impressed e. m. f. is 45°, and the damped impedance is $\sqrt{2}R$. With the other instruments tested, the equivalence of damped reactance and damped resistance was not obtained; but, as may be seen by reference to Figure 3, the curves of damped reactance and damped resistance for the watch-case instrument run nearly parallel and within 10 ohms of each other, for a considerable range of frequencies.

It would be interesting to discuss the relations expressed in equations (1), (2), (3), and (4). Since, however, at this time the primary purpose of the writers is to present an account of the effects of the motion of the diaphragm in modifying the resistance and reactance of the telephone receivers, a further discussion of the relations (1) to (4) will be deferred.

The Effects of Motion of the Diaphragm on the Resistance and Reactance of the Receivers.— As stated in the introduction, the motion of the diaphragm of a telephone receiver has a marked effect on its resistance and reactance. This effect is best shown by subtracting the damped resistance from the free resistance, and the damped reactance from the free reactance and plotting the differences, called respectively *motional resistance* and *motional reactance*, against the frequency in radians per second (angular velocity). This is done in Figures 4, 5, and 6. Taking Figure 4 plotted from Table IV obtained with the watch-case receiver, as typical, it will be seen that the Figure contains curves of motional resistance, motional reactance, motional power, and phase angle of motional impedance, marked respectively *Resistance, Reactance, Power* and *Phase.* These quantities are all plotted against angular velocity. The black dots are observed points, and the circles are computed values, or derived values. Beginning with the resistance curve, and remembering that this curve represents the excess of free resistance over damped resistance, that is to say, the effect of the motion, it will be seen that, starting at a value slightly below zero at 2834 radians per second, the increment of resistance due to motion (motional resistance) increases up to 23 ohms at angular velocity 5674, then descends rapidly to minus 25 ohms at angular velocity 5938 and then increases again toward zero. The motion of the diaphragm markedly increases the resistance at certain frequencies and markedly decreases it at other frequencies. The formulas for computing the motional resistance values are given under heading V below.

Next, let us examine the motional reactance curve. The effect of the motion of the diaphragm is chiefly to decrease the reactance so that the free reactance is less than the damped reactance, giving usually a negative motional reactance, amounting to — 44.7 ohms at angular velocity 5800. The motional reactance is not always negative but shows small positive values in the neighborhood of angular velocities 4500 and 7000.

The resemblance of the motional resistance curves and the motional reactance curve of Figures 4, 5, and 6 to the curves of optical index of refraction and optical absorption plotted against frequency, in the neighboring of an absorption band, will at once strike the attention of the reader familiar with theoretical optics. A difference, however, exists on account of the hysteretic behavior of the iron in the telephone theory, as will be pointed out in the treatment under heading V below.

Effect of Motion of Diaphragm on Draft of Power.— Attention is next directed to the curve marked *Power* in Figure 4. This curve shows the excess of power sent into the telephone when freely vibrating over the power sent into it under the same impressed e. m. f. when damped. The excess of power (i. e. *motional power*) is plotted in microwatts against angular velocity of impressed e. m. f., and is seen to be different for different angular velocities corresponding to different frequencies. The maximum of motional power is in the neighborhood of angular velocity 5820 radians per second, and this is the period of the diaphragm, as is shown later by other methods of analyzing the data. The impressed e. m. f. in this experiment was maintained throughout at 0.3 effective volt.

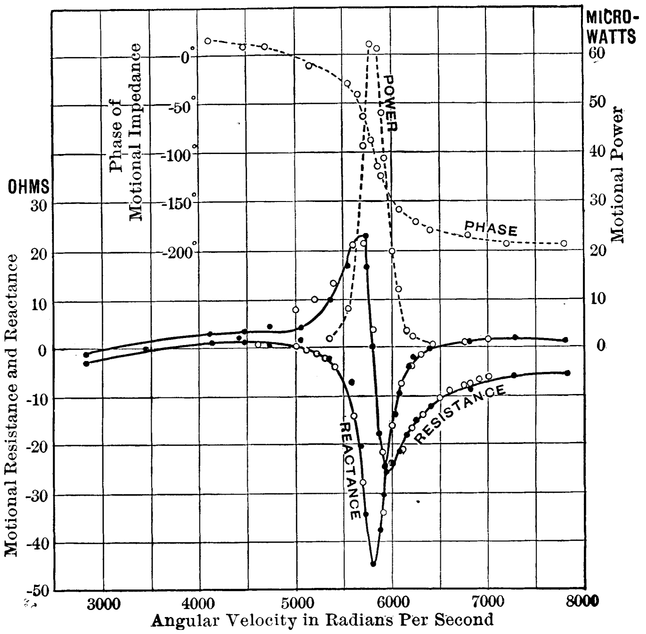

FIGURE 4. Curves of motional resistance, reactance, power, and phase, plotted against angular velocity, for watch-case receiver at 0.3 volt. Dots, observed; circles calculated.

The method of obtaining the motional power curve was as follows: Table III contains measurements of resistance and reactance of this receiver at different frequencies both while free and while damped. The square root of the sum of the squares of resistance and reactance gives directly the impedance. Dividing the impedance into the e. m. f. gives the effective current. The square of the free effective current multiplied by the free resistance gives the free power. Likewise, the square of the damped current multiplied by the damped resistance gives the damped power. The free power minus the damped power gives the *motional power*. These are tabulated for two receivers, for three series of measurements, in Tables VIII, IX and X.

It is not necessarily true that all of the motional power goes into energy of motion. The term means merely the excess of input when sounding over input when damped. As a matter of fact, the motional power is negative at some frequencies, as is shown in some of the curves

TABLE VII.

Showing Product of R and L for Bipolar Receiver *Rb* with 0.42 Volt at Terminals.

Frequency Cycles per Second. *n*.	Damped Resistance *R*. Ohms.	Damped Inductance *L*. Henry.	*RL*.	Departure. Per cent.
766	174	.0357	6.21	.0
778	175	.0354	6.20	.1
792	176	.0356	6.26	.8
805	177	.0351	6.22	.1
822	181	.0347	6.28	1.0
836	181	.0342	6.19	.3
853	183	.0342	6.26	.8
870	185	.0337	6.24	.5
890	186	.0335	6.23	.2
754	175	.0362	6.33	2.0
744	171.5	.0364	6.24	.5
733	170	.0369	6.26	.8
722	172	.0372	6.20	.1
710	167.5	.0373	6.25	.7
704	168	.0376	6.31	1.6
548	145.7	.0419	6.11	1.6
428	128.5	.0463	6.20	.1
2464	306.5	.0192	5.90	5.0
Average			6.21	

(e. g. Figures 5 and 6). Always, however, at consonance of the impressed e. m. f. with the period of the diaphragm, the motional power has a large positive value, which is no doubt correlative with the large amount of sound produced under this condition. With the receiver

TABLE VIII.

Values of Power. Bell Bipolar *Rb* at 0.3 Volt.

Frequency.		Power in Microwatts.		
n, Cycles per Second.	ω, Radians per Second.	Free.	Damped.	Motional.
440	2760	325.1	308.7	16.4
512	3220	301.4	303	− 1.6
600	3770	290.5	297.0	− 6.5
670	4210	274.2	283.2	− 9.0
704	4420	270.0	275.0	− 5.0
720	4520	272.7	271.8	.9
744	4660	282.0	266.2	15.8
754	4730	288.8	265.2	23.6
770	4830	304.5	261.0	43.5
778	4880	373.9	262.4	111.5
780	4900	356.9	266.2	90.7
790	4960	438.5	259.2	179.3
792	4970	436.0	259.8	176.2
792	4970	433.0	270.0	163.0
792	4970	423.3	259.2	164.1
793	4975	438.0	260.2	177.8
794	4980	428.0	259.0	169.0
804	5050	364.3	259.2	105.1
808	5080	422.2	257.3	164.9
822	5160	310.4	256.7	53.7
824	5180	371.5	254.0	117.5
824	5180	365.9	250.0	115.9
826	5190	352.0	253.0	99.0
826	5190	376.5	253.0	123.5
832	5220	309.0	260.0	49.0
840	5280	276.7	253.8	22.9
868	5450	269.2	230.6	38.6
890	5590	257.1	241.9	15.2
892	5600	257.1	244.2	12.9
912	5730	244.3	233.3	11.0
940	5900	244.2	239.6	5.6
1000	6283	222.2	220.1	2.1
1060	6650	215.1	213.0	2.1
1152	7250	192.0	190.5	1.5
1248	7850	195.7	197.5	− 1.8
1648	10350	177.4	187.3	− 9.9
2468	15500	153.3	152.1	1.2

giving the curves of Figure 4 (the bipolar watch-case — cf. Table X) the motional power at resonance is 62 microwatts, which is 20% of the total free input (309 microwatts) and 25% of the total input at the same voltage with the diaphragm damped (247 microwatts). In the case of the bipolar Bell receiver R_b at 0.3 volt (see Figure 5 and Table VIII) the motional power at resonance was 179 microwatts, which is 41% of the free input and 69% of the damped input. At 0.42 volt with receiver R_b (see Figure 6 and Table IX) the motional power was 338 microwatts at resonance, amounting to 40% of the total power input with diaphragm free and to 68% of the power input under the same e. m. f. with the diaphragm damped. That is to say, if one holds his finger on the diaphragm so as to damp it, and measures the power supplied to this receiver at 0.42 volt, the frequency being resonant with the period of the diaphragm, and then takes his finger off, the telephone emits a loud sound and the power input jumps up 68%.

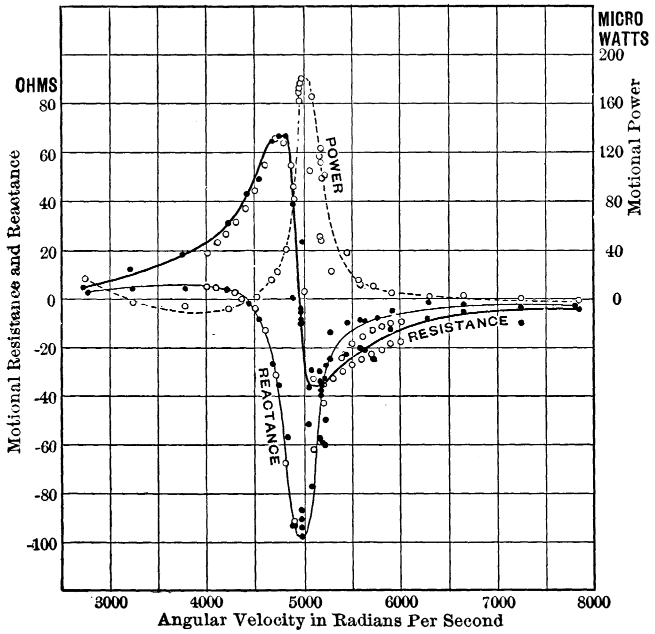

Figure 5. Curves of motional resistance, reactance, and power, *vs.* angular velocity, for Bell bipolar, with 0.3 volt. Dots, observed, circles calculated.

An examination of the curves of Figures 4, 5, and 6 shows how this occurs. The effective resistance of the receiver, with the e. m. f. at resonance, is not very different when it is sounding and when it is damped; that is, the motional resistance is nearly but not quite zero. What causes the large consumption of power at the resonant frequency is the low value of the effective inductive reactance of the receiver at this frequency and the consequent large draft of current from the source. As we go away from the frequency of e. m. f. resonant with the period of the diaphragm, the motional power consumption may be due either to excess of free resistance over damped resistance or to the excess of free current incidental to a decrease of inductance by the motion.

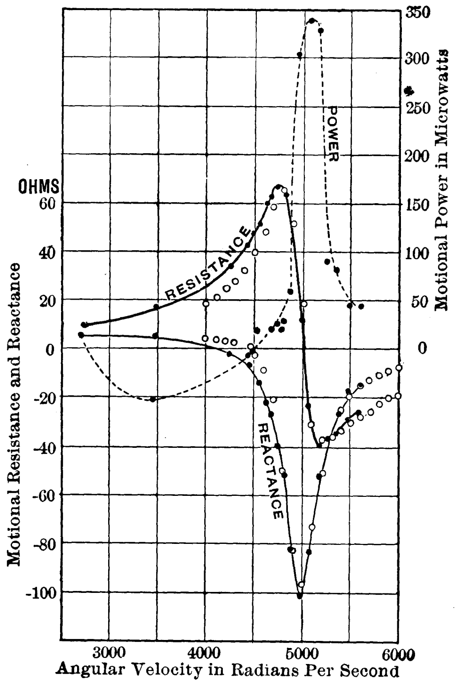

FIGURE 6. Same as Figure 5, but with 0.42 volt at terminals of Bell bipolar. Power points, calculated; other dots, observed; circles, calculated.

Effect of Motion on Phase.— The phase angle of the motional impedance for the watch-case receiver is shown in the curve marked

Phase in Figure 4. The angles plotted in this curve were obtained by taking the antitangent of the ratio of motional reactance to motional resistance and plotting the result, which is phase of motional impedance, against angular velocity. The meaning of this phase angle will be made plainer in the discussion of the circular graphs to follow.

TABLE IX.

VALUES OF POWER. BELL BIPOLAR R_b AT 0.42 VOLT.

Frequency.		Power in Microwatts.		
n, Cycles per Second.	ω, Radians per Second.	Free.	Damped.	Motional.
428	2690	677	666	11
548	3446	556	609	− 53
704	4425	520	530	− 10
710	4468	527	528	− 1
722	4540	539	520	19
733	4610	539	518	21
744	4680	546	518	28
754	4740	538	518	20
766	4810	541	512	29
778	4885	570	511	59
792	4976	800	496	304
805	5060	833	495	338
822	5170	825	493	332
836	5253	587	495	92
853	5354	564	481	83
870	5470	524	478	46
890	5595	518	471	47

Application to Sound Experiments.—It may be noted, in passing, that the effect of the reaction of the motion of the diaphram in modifying the electrical properties of the telephone receiver is of importance in experiments on sound, where an electrically driven tuning fork or telephone is used as the source of sound, because the power consumed in producing the sound may change with the change of the stationary sound-wave system in the room. In our experiments, the sound emitted from the test telephone was reflected from the various walls of the room and formed a stationary system with nodes and loops at various parts of the room. As an assistant walked about the room while the measurements were being made, it was found that the bridge, previously balanced, was thrown successively in and out of balance as the reflection and absorption of the assistant's clothing

TABLE X.

VALUES OF POWER. WATCH-CASE RECEIVER AT 0.3 VOLT.

Frequency.		Power in Microwatts.		
n, Cycles per Second.	Radians per Second.	Free.	Damped.	Motional.
451	2834	396	376	20
550	3456	328	330	− 2
653	4102	302	299	3
702	4410	276	280	− 4
712	4474	284	285	− 1
754	4738	274	274	0
804	5052	264	268	− 4
849.5	5340	258	256	2
884	5554	259	251	8
903	5674	271	250	21
913	5736	290	249	41
923	5800	309	247	62
934	5868	305	244	61
940	5906	291	243	48
945	5938	282	243	39
957	6014	261	241	20
968	6082	252	240	12
980	6158	244	241	3
993	6240	241.5	239	2.5
1020	6408	235	234	1
1084	6812	228	227	1
1157	7270	222	219	3
1250	7854	214	212	2
1846	11600	174	172	2

changed the stationary sound system when he walked through the room. Professor Sabine, in some experiments not yet published, had previously noticed the effect of a shift in the stationary wave system in modifying the draft of power by an electrically-driven tuning fork kept vibrating at constant amplitude. In accordance with the present experiments and as Professor Sabine previously suggested in conversation with one of the writers, the phenomenon is seen to have its explanation in the change of resistance and reactance of the coil of the fork due to the variously affected motion of the fork.

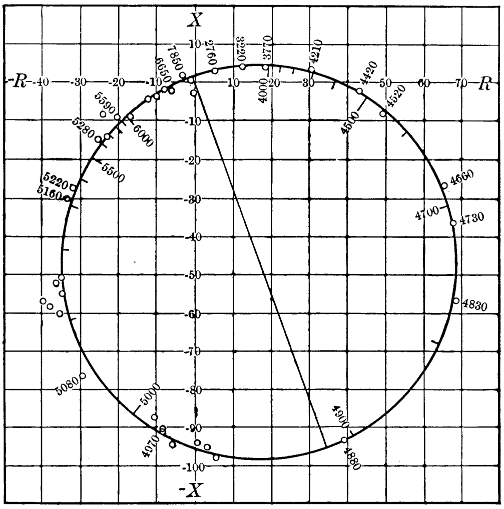

Figure 7. Circular graph for bipolar Bell receiver R_b with 0.3 volt at terminals. Diameter 103 ohms. Depression angle (2β) 70.5°. $\omega_0 = 4885$ radians per sec. $\Delta = 200$. Small circles observed. Internal ring numbers computed.

Circular Graphs of Motional Impedance.— A very interesting result is obtained by plotting the motional reactance of a telephone as ordinate against the motional resistance as abscissa. The result is a point on the $R\ X$ plane, and the point is different for different values of the angular velocity used in the measurement. The locus of this point, as the angular velocity is varied, is a circle passing through the origin; that is, through the point of zero motional resistance and zero motional reactance. Stated otherwise, if the motional impedance, $(R' - R) + j(X' - X)$, is plotted vectorially from a point as origin, the vector for any given frequency is the cord of a circle through the point. As the angular velocity of the impressed e. m. f. increases from zero to infinity, the free end of the vector impedance passes once around the circle. Circular graphs of this character are plotted in Figures 7 to 11 for different instruments or for different values of impressed

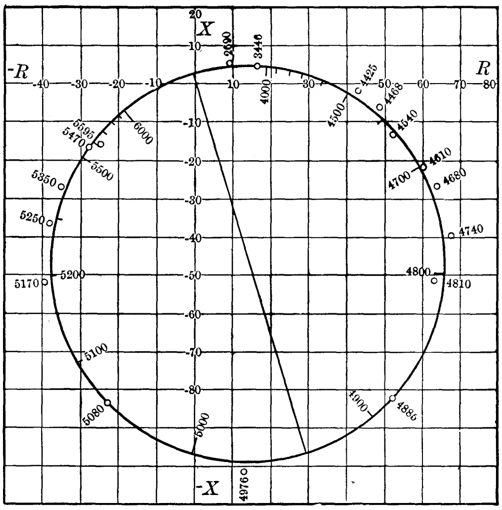

Figure 8. Circular graph for bipolar Bell receiver R_b with 0.42 volt at terminals. Diameter 103.5 ohms. Depression angle (2β), 73°; $\omega_0 = 4940$ radians per second; $\Delta = 200$. Small circles observed. Internal ring numbers computed.

e. m. f. These several circular graphs are on different scales, and are summarized to the same scale in Figure 12.

On each of the circular graphs, all of the observed points, for a given receiver with a given impressed e. m. f., are plotted as small circles. The measured angular velocities of the impressed e. m. f. for the observed points are printed at the *outside* of the circular locus. To avoid crowding, not all of the points are designated with angular velocities, and in the selection of the points to bear numerical designation of angular velocity, those points were chosen, for which the measurements were made with especial precaution as to voltage and frequency.

The numbers placed *inside* of each of the circular loci are computed values for the theoretical distribution of angular velocities around

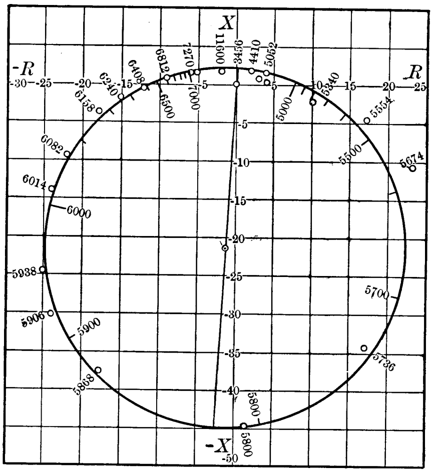

FIGURE 9. Circular graph for watch-case receiver with 0.3 volt at terminals. Diameter 47 ohms. Depression angle (2β) 93°. $\omega_0 = 5820$ radians per second. $\Delta = 150$. Small circles observed. Internal ring numbers computed.

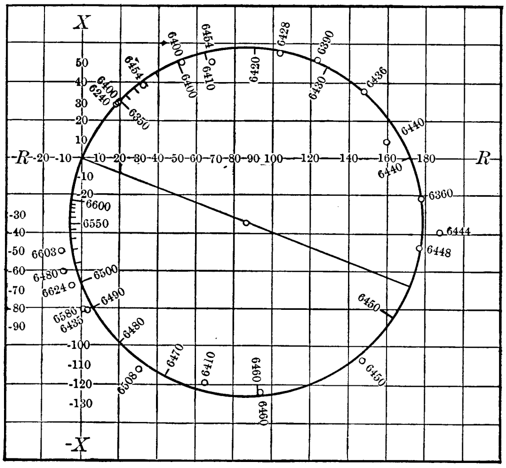

FIGURE 10. Circular graph for experimental monopolar receiver, with 0.3 volt at terminals. Diameter 185.4 ohms. Depression angle (2β) 21.5°. $\omega_0 = 6448$ radians per second. $\Delta = 20$. Small circles observed. Internal ring numbers computed.

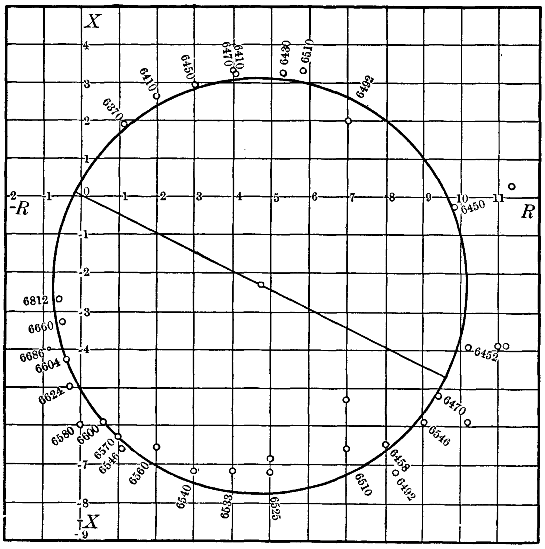

FIGURE 11. Circular graph for experimental bipolar receiver, with one volt at terminals. Diameter 10.9 ohms. Depression angle (2β) 26.5°. ω_0 = 6465?; Δ = 20?.

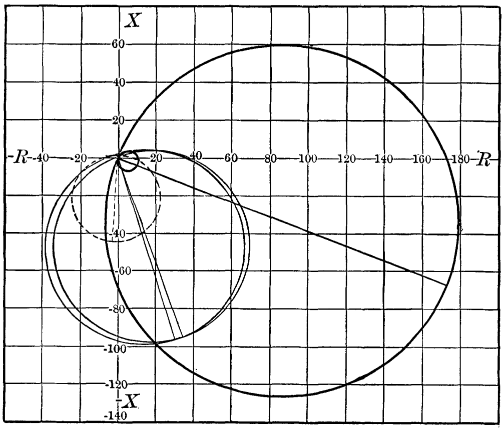

FIGURE 12. Circular graphs collected to one and the same scale. The largest circle and the smallest circle are graphs with experimental receivers. Two light circles near together are graphs with bipolar Bell receiver. Broken circle is graph with watch-case.

the circular graphs. The formulas by which this distribution has been computed are derived under heading V below.

The quantities of theoretical and practical importance in these circular graphs are:

1. The length of diameter of the circular graph for a particular receiver.
2. The dip of the diameter below the axis of resistance.
3. The rate of change of angular velocities around the circle.
4. The angular velocity at the end point of the diameter, remote from the origin, and
5. The impedance at this point.

The significance of these several quantities will appear in connection with the discussion of the theory of the problem, which follows.

V. THEORY OF THE REACTIVE EFFECTS OF MOTION OF THE DIAPHRAGM ON THE ELECTRICAL CONSTANTS OF THE RECEIVER.

An exact treatment of the electrical properties of a coil containing a magnetic core in proximity to a moving magnetic membrane offers great difficulty. If, however, we confine our attention to terms of the first order, we can obtain a sufficiently close approximation to a solution, to permit an interpretation of the preceding experimental results.

Assumptions Regarding Mechanical Magnitudes.— To this end we shall assume, so far as concerns the fundamental mode of vibration of the diaphragm,

(1) That the elastic restoring force of the diaphragm is all concentrated at the center of the diaphragm, and is proportional to the displacement;

(2) That the motion is opposed by a frictional force proportional to the velocity and also concentrated at the center of the diaphragm; and

(3) That the actual distributed mass of the diaphragm may be replaced by an equivalent mass concentrated at the center of the diaphragm.

Motion of the Center of the Diaphragm under a Pull Maintained Sinusoidal.— As a first step toward the solution, let us assume the diaphragm to be solicited by a force which is maintained sinusoidal; then (*cf.* Figure 13)

$$sx + r\dot{x} + m\ddot{x} = f = F\epsilon^{j\omega t} \qquad {}^{2}\text{dynes} \angle \qquad (5)$$

---

[2] The sign $\angle$ following the unit indicates that the equation should be interpreted vectorially, or in complex quantities.

in which

x = the displacement of the effective mass of the diaphragm from its position of rest (cm.),

$\dot{x}$ = the displacement velocity (cm/sec),

$\ddot{x}$ = the displacement acceleration (cm/sec^2),

s = the elastic force per unit displacement (dynes/cm),

r = the resisting force per unit velocity (dynes per cm. per sec.)

f = $F\epsilon^{j\omega t}$, the impressed moving force measured in the direction of x toward the poles (dynes),

ω = $2\pi n$, the angular velocity of the impressed force (radians per second), and

n = the frequency of the impressed sinusoidal force (cycles per second).

$j = \sqrt{-1}$

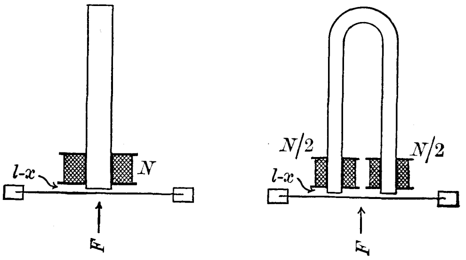

FIGURE 13. Diagram of receivers. N is number of turns. F is force on diaphragm with direction of arrow; l is normal gap length; and x displacement toward poles.

The solution of equation (5) for velocity of displacement, after a steady state has been attained, is well known, and may be written in the form

$$\dot{x} = \frac{f}{r + j\left(m\omega - \frac{s}{\omega}\right)} = \frac{f}{z} = \frac{F\epsilon^{j\omega t}}{z} \quad \text{cm/sec} \angle \qquad (6)$$

in which

$$z = r + j\left(m\omega - \frac{s}{\omega}\right) \qquad \text{dyne sec/cm} \angle \qquad (7)$$

The quantity z may be called "vector mechanical impedance" from its analogy to vector electrical impedance.

We may further write for abbreviation

$$|z| = \sqrt{r^2 + \left(m\omega - \frac{s}{\omega}\right)^2} \quad \text{dyne sec/cm} \qquad (8)$$

and

$$a = tan^{-1}\left\{\frac{m\omega - \frac{s}{\omega}}{r}\right\} \quad \text{radians} \qquad (9)$$

The quantities entering in the above equations, and their analogous electrical quantities, are tabulated below.

Mechanical Quantity.		Electrical Quantity.	
Velocity of displacement	$\dot{x}$	i	Current
Mechanical force	f	e	Electromotive force
Resistance (i. e., force of resistance per unit velocity)	r	R	Resistance (i. e., e. m. f. of resistance per unit current)
Effective mass (i. e., force per unit time-rate of change of velocity)	m	L	Self-inductance (i. e., e. m. f. per unit time-rate of change of current)
Elastic force per unit displacement (i. e., per unit time-integral of velocity)	s	$1/C$	Reciprocal of capacity (i. e., e. m. f. per unit time-integral of current)
Vector mechanical impedance	z	Z	Vector electrical impedance
Mechanical Impedance	$\|z\|$	$\|Z\|$	Electrical impedance
Mechanical phase-angle	a	θ	Electrical phase angle
Mechanical inertia reactance	$m\omega$	$L\omega$	Electrical inductive reactance
Mechanical elastic reactance	s/ω	$(1/C\omega)$	Electrical capacity reactance

Circular Graph of Velocity.— By equation (6) $\dot{x}$ is seen to be sinusoidal, with amplitude $\frac{F}{|z|}$ and lagging by an angle a behind the impressed force. A geometrical representation of the amplitude and phase of $\dot{x}$ is given in Figure 14. In the left hand part of the figure Op is a representation of the vector mechanical impedance z. As ω changes from zero to infinity, the point p moves along the straight line Xp from minus infinity to plus infinity, parallel to OY. In the right hand part of the figure, the circle is the vector graph of F/z, which is given in magnitude and direction by OP. This circle is obtained by taking the reciprocal of the straight line locus of the vector

z and multiplying the reciprocal by F, which gives a circle of diameter F/r symmetrically disposed with reference to the axis of reals.

The use of this circle is as follows. For a given value of ω find the angle α by equation (9) and lay off this angle negatively at O; then the length of the chord OP of the circle gives the amplitude $\frac{F}{|z|}$ of x, for the given ω, and the angle is the angle of lag of $\dot{x}$ behind the impressed force. As ω changes from zero to infinity, the point P moves negatively once around the circle from O, through X, back to O.

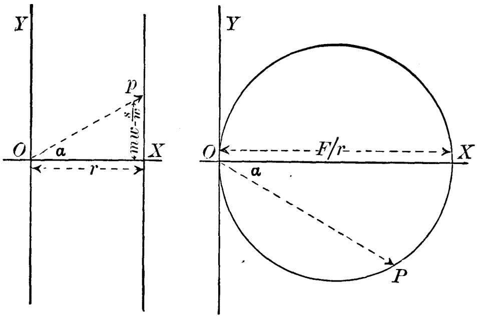

FIGURE 14. Left straight line graph of z. Right, circular graph of F/z.

Magnetic Flux as Dependent on Current and Mechanical Displacement.— In the problem under consideration, the pull f acting on the diaphragm is determined by the magnetic flux through the air gap, or air gaps, of the receiver. If ϕ is the mean flux through the active part of the magnetic circuit, we have, for a bipolar receiver, approximately

$$\phi = \frac{\mathfrak{F}}{\mathfrak{R}} = \frac{\mathfrak{F}_0 + 4\pi Ni}{\mathfrak{R}_0 + \frac{2(l-x)}{S}} = \mathfrak{B} S \quad \text{maxwells} \quad (10)$$

and for a monopolar receiver, approximately

$$\phi = \frac{\mathfrak{F}}{\mathfrak{R}} = \frac{\mathfrak{F}_0 + 4\pi Ni}{\mathfrak{R}_0 + \frac{l-x}{S}} \quad \text{maxwells} \quad (11)$$

in which

$\mathfrak{F}$ = the total m. m. f. due to the permanent magnet and to the current i in the coils (gilberts),

$\mathfrak{F}_0$ = m. m. f. due to permanent magnet alone (gilberts),

$\mathfrak{R}$ = total reluctance of the magnetic circuit (oersteds),
$\mathfrak{R}_o$ = reluctance of circuit exclusive of that of the gaps (oersteds),
N = total number of turns in the receiver coils,
i = instantaneous current in the coils assumed to vary sinusoidally, or according to the real part of $I\epsilon^{j\omega t}$ (absamperes),
l = normal air-gap between poles and diaphragm (cm.),
$\mathfrak{B}$ = mean flux density in the air gap (gausses), and
S = area of one gap (cm^2).

The Equations of Current and Motion.—We can now express the pull on the diaphragm in terms of the flux. It is a well known fact, which may be derived from energy relations, that the pull on the diaphragm is

$$f = \frac{\phi^2}{4\pi S} \text{ for a bipolar receiver} \qquad \text{dynes} \quad (12)$$

and

$$f = \frac{\phi^2}{8\pi S} \text{ for a monopolar receiver} \qquad \text{dynes} \quad (13)$$

If now f_i is used to denote the part of the pull due to the current i, and if this is small in comparison with the pull due to the permanent magnet, we may write

$$f_i = \frac{df}{di}\, i \qquad \text{dynes} \angle \quad (14)$$

which by substitution from equations (12) and (10) becomes

$$\begin{aligned} f_i &= \frac{2\phi}{4\pi S}\,\frac{d\phi}{di}\, i \\ &= \frac{2N\phi}{\mathfrak{R}S}\, i = \frac{2N\mathfrak{B}_o}{\mathfrak{R}}\, i \text{ for a bipolar} \qquad \text{dynes} \angle \quad (15) \end{aligned}$$

and

$$f_i = \frac{N\mathfrak{B}_o}{\mathfrak{R}}\, i \text{ for a monopolar.} \qquad \text{dynes} \angle \quad (16)$$

In equations (15) and (16) $\mathfrak{B}_o$ has been substituted for $\mathfrak{B}$, since the increment in $\mathfrak{B}_o$ due to i, when multiplied by i, is assumed to be a second order effect.

In order to avoid carrying through separate discussions for the bipolar and for the monopolar receiver, and in order to simplify the equations, let us write

$$A = \frac{2N\mathfrak{B}_0}{\mathfrak{R}} \text{ for the bipolar receiver, and} \qquad \text{dynes/absampere} \quad (17)$$

$$A = \frac{N\mathfrak{B}_0}{\mathfrak{R}} \text{ for the monopolar receiver;} \qquad \text{dynes/absampere} \quad (18)$$

then, for either instrument,

$$f_i = Ai \qquad \text{dynes } \angle \quad (19).$$

Equations (15), (16) and (19) assume that the pull on the diaphragm due to i is in the phase with i; but with hysteresis and eddy currents present, the electromagnetic force will lag[3] behind the current i by an angle β_1; whence the force on the diaphragm due to the current i becomes, by eq. (19),

$$f_i = A\,i\,\overline{\backslash\beta_1} \qquad \text{dynes } \angle \quad (20)$$

Consequently, by equation (6),

$$\dot{x} = \frac{A\,i\,\overline{\backslash\beta_1}}{z} \qquad \text{cm/sec } \angle \quad (21)$$

The e. m. f. induced in the coils by the motion of the diaphragm will be, in the absence of hysteresis,

$$e_x = N\frac{d\phi}{dt} = N\frac{\partial\phi}{\partial x}\dot{x} \qquad \text{abvolts, } \angle \quad (22)$$

and by differentiating equation (10) or (11), equation (22) gives to a first approximation

$$e_x = \frac{2N\mathfrak{B}_0\dot{x}}{\mathfrak{R}} = A\dot{x} \qquad \text{abvolts, } \angle \quad (22a)$$

and by substitution from equation (21)

$$e_x = \frac{A^2 i\,\overline{\backslash\beta_1}}{z}. \qquad \text{abvolts } \angle \quad (23)$$

However, it should be noted that there is also a hysteretic lag of flux with change of gap, and this will cause the induced e. m. f. to lag by a certain angle β_2 behind $\dot{x}$, so that equation (23) should be changed to

$$e_x = \frac{A^2 i\,\overline{\backslash\beta_1 + \beta_2}}{z} \qquad \text{abvolts } \angle \quad (24)$$

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[3] On the question of Constancy of β_1 see VI below.

If L and R are the inductance and resistance of the receiver when damped, the impedance of the damped receiver will be

$$Z = R + jL\omega \qquad \text{absohms} \angle \quad (25)$$

and if e is the instantaneous value of the impressed e. m. f. of the type $E\epsilon^{j\omega t}$, we shall have

$$e = iZ \qquad \text{abvolts} \angle \quad (26)$$

But owing to the influence of the e. m. f. of motion, the last equation becomes modified to

$$e - e_x = iZ \qquad \text{abvolts} \angle \quad (27)$$

or

$$e = iZ + e_x \qquad \text{abvolts} \angle \quad (28)$$

That is by equation (24)

$$e = i\left\{ Z + \frac{A^2}{z} \backslash\overline{\beta_1 + \beta_2} \right\} = iZ' \quad \text{abvolts} \angle \quad (29)$$

where Z' is the free impedance of the receiver.

This means that the impedance of the receiver has become increased, through the vibration of the diaphragm, by *a motional impedance:*

$$Z' - Z = \frac{A^2}{z} \backslash\overline{\beta_1 + \beta_2} \qquad \text{absohms} \angle \quad (30)$$

This motional impedance, being the reciprocal of the vector equation of a straight line with ω as variable, is a circle for variable ω, and has a diameter $\frac{A^2}{r}$, depressed below the axis of reals by an angle $\backslash\overline{\beta_1 + \beta_2}$.

As to the relative values of β_1 and β_2 it seems reasonable that whether the change of flux of a circuit is caused by a small change of current, changing the m. m. f., or by a small change of gap-length, changing the reluctance, the angle of lag of flux behind the cause is the same; that is $\beta_2 = \beta_1 = \beta$ (Say). This is borne out by one of our experiments to be described below (see VI). With this equivalence substituted in equations above, we obtain,

$$Z' - Z = \frac{A^2}{z} \backslash\overline{2\beta} \qquad \text{absohms} \angle \quad (31)$$

Consequently, if we vary ω from 0 to $+\infty$, keeping the impressed e. m. f. and all other quantities constant, the motional impedance $Z' - Z$ has a circular graph through the origin, with its principal diameter of length $\frac{A^2}{r}$ depressed 2β below the axis of reals. Equation (31) is the theoretical equation to the circular graphs of Figures 7 to 12.

Replacing the vector z of equation (31) by its absolute value $|z|$ and angle α, we have

$$Z' - Z = \frac{A^2}{|z|} \overline{\backslash 2\beta + \alpha} \qquad \text{absohms} \angle \qquad (32)$$

Equation (32) may be analysed into

$$R' - R = \frac{A^2}{|z|} \cos(2\beta + \alpha) \qquad \text{absohms} \qquad (33)$$

$$X' - X = \frac{-A^2}{|z|} \sin(2\beta + \alpha) \qquad \text{absohms} \qquad (34)$$

in which

$$|z| = \sqrt{r^2 + \left(m\omega - \frac{s}{\omega}\right)^2}, \qquad \text{absohms} \qquad (35)$$

and

$$\alpha = \tan^{-1}\left\{\frac{m\omega - \frac{s}{\omega}}{r}\right\} \qquad \text{radians} \qquad (36)$$

are functions of ω. The quantity A, involving $\mathfrak{B}_0$ and $\mathfrak{R}$, might be expected to vary with variation of ω, but an examination of the experimental results shows that, with the excitations employed, not much error is introduced by considering A and also β independent of ω.

Equations (32), (33), and (34) are in convenient form for computation, and permit an easy determination of some of the important mechanical constants of the diaphragm.

For example, if we let ω_0 be the angular velocity of impressed mechanical force for which the sustained vibration of the diaphragm is in resonance, we see from equation (6) above that

$$\omega_0 = \sqrt{\frac{s}{m}} \qquad \text{radians/sec.} \qquad (37)$$

Now, if ω, the angular velocity of the impressed electromotive force in the telephone circuit, is equal to $\omega_0 = \sqrt{\frac{s}{m}}$, it is seen by equation (36) that α becomes zero; hence the value of ω, which in the experimental circular graphs of Figures 7 to 10 lies at the remote end of the principal diameter is the $\omega = \omega_0$ for which the diaphragm in sustained vibration is resonant. This gives a simple and accurate method of determining ω_0 for a telephone diaphragm.[4]

Again, let Δ be the logarithmic decrement per second of the diaphram, if vibrating under no external force, then by the theory of elasticity,

$$\Delta = \frac{r}{2m}, \qquad \text{numeric/sec.} \quad (38)$$

whence from (36)

$$\omega \tan \alpha = \frac{\omega^2 - \omega_0^2}{2\Delta} \qquad \text{numeric/sec.} \quad (39)$$

Differentiating (39) with respect to α, we obtain

$$\frac{d\omega}{d\alpha} \tan \alpha + \omega \sec^2 \alpha = \frac{\omega}{\Delta} \frac{d\omega}{d\alpha} \quad \text{numeric/sec.} \quad (40)$$

and if

$$\alpha = 0,$$

$$\left[\frac{d\omega}{d\alpha}\right]_0 = \Delta; \qquad \text{numeric/sec.} \quad (41)$$

That is, in the experimental circular graphs, the rate of change of ω with change of α, at the remote end of the principal diameter, is the logarithmic decrement per second of the diaphragm. This quantity cannot, however, be obtained with the precision with which ω can be obtained.

Another method of obtaining Δ is by taking the values of ω_1 and ω_2 which lie respectively 45° below and 45° above the principle diameter,— these angles being measured at the origin, not at the center. For these points $\tan \alpha$ is respectively $+ 1$ and $- 1$; whence from (39)

$$2\Delta\omega_1 = \omega_1^2 - \omega_0^2$$
$$-2\Delta\omega_2 = \omega_2^2 - \omega_0^2$$

and by subtraction and division by $2(\omega_1 + \omega_2)$,

$$\Delta = \frac{\omega_1 - \omega_2}{2} \qquad \text{numeric/sec.} \quad (42)$$

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[4] For another method of finding ω_0 from the humming tone of a telephone receiver, see a paper by A. E. Kennelly and W. L. Upson, Proc. Am. Phil. Soc., 1908, "The Humming Telephone."

Thus we have methods of determining both ω_0 and Δ. The experiments, on the other hand, do not permit a direct determination of the quantities m, r, and s; but it would seem that by adding a known mass, as a small load, to the center of the diaphragm and repeating the series of measurements, these quantities should be capable of determination.

VI. Comparison of Experimental Results with Theory.

An examination of the experimental results with the aid of the theory above developed gives the following results, which may be called the characteristics of the several receivers (Table XI):

TABLE XI.

Summary of Characteristics of Receivers.

	Bipolar Bell R_b at 0.3 Volt.	Bipolar Bell R_b at 0.42 Volt.	Watch-case at 0.3 Volt.	Experimental Monopolar at 0.3 Volt.	Experimental Bipolar at 1 Volt.
Diameter of motional impedance circle in ohms.	103	103.5	47	185.4	10.9
Depression angle (2β) in degrees	70.5	73	93	21.5	26.5
ω_0 in radians per second	4885	4940	5220	6448	6465 ?
Log. dec. per second. (Δ)	200	200	150	20.	20 ?

The method of obtaining these characteristics was as follows: The circular graphs of Figures 7 to 11 were plotted. The diameter of the motional impedance circle and the angle of depression of this diameter below the axis of $R'-R$ could be measured off at once on the diagram. The value of ω at the free end point of the diameter could also be read or obtained by interpolation; this ω is the ω_0 of the diaphragm. The logarithmic decrement per second Δ could have been obtained by either of the two methods derived in the discussion of the theory, equations (41) or (42); but a third method was employed; namely, by the use of the more general equation (39), in which several values of α and the corresponding values of ω from the circular graphs were substituted, and the values of Δ so obtained were averaged.

Having now obtained the constants of Table XI, the theoretical distribution of angular velocities around each of the circular graphs of Figures 7 to 11 were calculated by equation (39), and these theoretical values are designated by numerals on the inside of the circular graphs.

The values of $R'-R$ and of $X'-X$ corresponding to these theoretical values of ω were then plotted as the circles on the rectangular graphs of motional resistance and motional reactance in Figures 4, 5, and 6. It is seen that the agreement of the computed and observed points in these Figures 4 to 6, while not exact throughout the entire range, is yet sufficiently good to show that the theory is essentially correct.

Another significant point in the theory is the interpretation we have given to the depression angle $2\,\beta$ of the circular graphs. We interpreted β to be the angle by which the magnetic flux lags behind the magnetizing current in the telephone receivers. To test this point, this angle of lag of magnetic flux behind magnetizing current was independently measured with the experimental bipolar receiver. This receiver had a separate secondary, or exploring, coil wound around the ends of its poles, near the diaphragm. The e. m. f. generated in this exploring coil is in phase with the time rate of change of flux; and the phase of this e. m. f. was compared with the phase of the alternating current in the exciting coils in two ways (1) by a three-voltmeter method and (2) by an alternating current potentiometer.

In the three-voltmeter method, a known resistance was put in series with the exciting coils, and one end of the exploring coil was connected to the point between the exciting coil and the known resistance. With the frequency and the e. m. f. about the telephone kept the same as that used in the bridge measurements (i. e., the e. m. f. of 1 volt, and the frequency near the resonant frequency of the diaphram) voltages were measured about the known resistance (20 ohms), about the exploring secondary, and about the two in series. These voltages, being small, were measured by a crystal rectifier in series with a galvanometer,[5] — the galvanometer and rectifier having been calibrated immediately before and after the experiment by an a. c. potentiometer operating at the frequency employed.

The readings of voltage were very consistent, and were as follows in a typical case:

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[5] G. W. Pierce: Phys. Review, **25**, p. 31, 1907; ibid., **28**, p. 153, 1909.

Voltage about 20 ohms = 0.129 volt
" about secondary = 0.125 "
" about both = 0.196 "
" about both with secondary reversed = 0.161 "

Substitution of the first three of these values in the formula for an obtuse-angle oblique triangle gives 79°, as the angle by which the secondary voltage leads the primary current. This is the angle by which the time derivative of the magnetic flux leads the primary current. The flux itself lags its time derivative by 90°, and therefore lags the primary current by $90° - 79° = 11°$.

Again, a substitution of the first, second, and fourth value of above table in the formula for an acute-angle oblique triangle gives for the flux lag angle the value 11.5°.

This angle of lag of flux behind the magnetizing current was found to be nearly independent of the frequency. To illustrate this, and as a further confirmation of the result obtained by the three-voltmeter method with the crystal rectifier and galvanometer as voltmeter, a second measurement was made by an entirely different method; namely, by the use of a Drysdale alternating-current potentiometer, with a 60-cycle current, and with a vibration galvanometer as indicating instrument. The method employed in this experiment consisted in first measuring the magnitude and phase of the primary current, and then the magnitude and phase of the voltage in the secondary winding. The difference between these two phases, subtracted from 90^0, gives the required angle of magnetic flux lag. Balance was in each case indicated by getting a zero deflection of the vibration galvanometer. This method gave 12.5° as the angle by which the flux in the telephone lags behind the magnetising current.

The three values obtained by direct measurement for the flux lag, which should be the angle β according to the theory above proposed, are 11°, 11.5°, and 12.5°; whereas half the depression angle, for this telephone, which, according to the theory, should also be the angle β, is 13.2°. The agreement is not as good as might be desired for a perfect confirmation of the proposed theory; but in view of the difficulty of measuring small angles of lag in circuits containing voltages of the order of 0.1 volt, and in view of the fact that the experimental telephone receiver constructed for this purpose had to be complicated by auxiliary secondary windings and also unfortunately had a diaphragm mounted in such a way as to have a very large temperature coefficient of vibration period, which rendered difficult an accurate determination of the points of the circular graph, the writers believe that the departure of a degree or two in the value of β, as obtained by direct measurement from its value as obtained by the circular graphs, is not unsatisfactory.

VII. Summary of Results.

1. The resistance and inductance of several telephone receivers were measured over a wide range of frequencies with their diaphragms both free and damped.
2. The damped resistance is approximately a quadratic function of the angular velocity of impressed e. m. f. (see equations (1) and (2)).
3. Although the damped resistance and the damped inductance both change with the frequency of e. m. f., their product is approximately constant, independent of the frequency, over a considerable portion of the range of audible frequencies (see eq. (3) and (4) and Table VII).
4. The damped reactance of one form of standard bipolar Bell receiver is approximately equal to its damped resistance, over a considerable range of frequency; so that the current lags the e. m. f. by 45° (see Figure 2).
5. The free resistance and reactance of telephone receivers go through marked changes with changes in frequency of constant e. m. f. in the neighborhood of the natural frequency of their diaphragms (*cf.* Figures 4–6).
6. The motional resistance and motional reactance (by which is meant excess of free resistance of reactance over damped resistance or reactance) conform accurately to certain simple laws as follow:
    - I. The motional reactance plotted as ordinates against the motional resistance as abscissas, as the frequency of constant impressed e. m. f. is changed from zero to infinity, gives a circular locus, with various interesting characteristics (*cf.* Figures 7–12).
    - II. The rectangular plots of motional reactance and motional resistance against angular velocity of constant impressed e. m. f. give curves somewhat analogous to the curves of index of refraction and absorption of light in an optical medium in the neighborhood of an absorption band (*cf.* Figures 4–6).
7. The power taken by a telephone receiver when sounding at 0.3 volt applied voltage may exceed by 68% the power taken from the

same e. m. f. when the diaphragm is damped (Figures 4–7 and Tables VIII–X).

8. A theoretical explanation of the phenomena is given, and computations are submitted in comparison of experiment and theory (Headings V and VI).

9. The vibration constants of the diaphragms of the several receivers are deduced and collected (Table XI).

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